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# Non-supersymmetric stable vacua of M-theory

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Abstract: We discuss the stability of non-supersymmetric compactifications of M-theory and string theory of the form $AdS \times X$, and their dual non-supersymmetric interacting conformal field theories. We argue that some of the difficulties in controlling $1/N$-corrections disappear in the cases that the large-N dual conformal field theory has no invariant marginal operators (and in some cases with no exactly marginal operators only). We provide several examples of such compactifications of M-theory down to $AdS_4$.

Keywords: M-Theory, Superstring Vacua, 1/N Expansion.

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## Contents



# 1. Introduction

A powerful tool in the study of large N limits of conformal field theories is their description in terms of certain string theory or M-theory vacua [2]. These vacua are of the form $AdS_p \times X_q$, where $X_q$ is a compact Einstein manifold and $p + q$ adds up to 10 or 11. A precise prescription of how to derive conformal dimensions and correlation functions from the supergravity side was given in [3, 4] and has been extensively developed since.

Of particular interest is the description of vacua that break supersymmetry completely. The interest is drawn from both field theoretic and M-theory points of view. From the field theoretic point of view, one would like to have some control over non-supersymmetric interacting conformal field theories as a tool for better understanding non-supersymmetric field theories in general. The existence of such conformal fixed points is well established [5] but only in the perturbative regime. There is no concrete proposal for a strongly coupled non-supersymmetric conformal field theory in $d > 2$.

The issue is even more interesting from the point of view of M-theory. The supergravity background that is dual to the large N limit of such conformal field theory will be a vacuum with no supersymmetry which is nevertheless stable. The understanding of stable non-supersymmetric vacua would be an important step forward in order for string theories to make contact with reality.

A procedure for obtaining non-supersymmetric supergravity vacua that may be dual to $d = 4$ non-supersymmetric conformal field theories has been discussed in [6] in the context of Type IIB string theory orbifolds (related issues have been studied subsequently in [7]). As discussed in that work, there might be a non-vanishing dilaton tadpole, which would destabilize the vacuum (unless the dilaton potential

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remains flat, as was suggested recently [8] for some cases). This implies that the true quantum vacuum may be quite far away in the space of vacua. One indeed expects that this would be a generic problem of orbifolds at weak string coupling. The phenomenon is the well-known Dine-Seiberg problem re-emerging in this context [9].

In this paper, we will discuss a construction in M-theory of vacua which break supersymmetry completely, but nevertheless overcome this specific problem: there is no candidate for a field which might develop a significant tadpole when taking into account possible $\frac{1}{N}$ corrections (although we do not know how to actually compute them in M-theory). The usefulness of M-theory for freezing moduli was already observed in [10]. We will focus primarily on compactification down to $AdS_4$.

In the following section, we will discuss an argument why a certain class of vacua of M-theory may be close to a stable one in a sense that will be made precise there. In section 3 we discuss a concrete set of non-supersymmetric vacua of M-theory of the form $AdS_4$ times an Einstein manifold $M_7$ which fall into this class.

## 2. A conformal field theory argument for stability

Suppose we are given an M-theory vacuum which is dual, as $N \to \infty$, to a conformal field theory. One can then argue that if the conformal field theory contains no invariant marginal operators then the effects of the tadpoles is to modify the true conformal fixed point as well as the true supergravity vacuum, whether supersymmetric or not, by effects that are only of order $1/N$.

The main assumption that goes into the argument is what we will call *calculability*. We will assume that given a supergravity vacuum at large $N$ limit there exists a well-defined calculational procedure such that this vacuum corresponds to the classical theory. This is a very natural assumption from the point of view of supergravity side but when translated to the conformal field theory side it becomes very powerful.

Since we do not know how to quantize such vacua of M-theory or string theories with Ramond-Ramond flux, we cannot compute the $\frac{1}{N}$ corrections. Nevertheless, the assumption of calculability can carry us some steps forward. The implication of this assumption is that gauge symmetries of the classical supergravity background can be broken only spontaneously, if at all, by the $\frac{1}{N}$ corrections. In that case, any supergravity field that is charged under the gauge symmetries would not have a linear tadpole (of the form $\mathcal{L}_{\mathrm{SUGRA}} = \cdots + \lambda^* \phi + \lambda \phi^* + \cdots$) since such a tadpole will necessarily break gauge symmetries explicitly. Since we cannot generate a linear tadpole for the charged fields, we can at most change their mass term (by $1/N$ corrections). This in turn will change their dimension around the conformal fixed point but will not destabilize it. Higher order corrections to these fields will not even change the dimension. Our only concern, therefore, are linear tadpoles for fields that are *neutral* under all the global symmetries of the dual conformal field theory, (i.e., the associated particles are not charged under the gauge symmetries of the classical supergravity background). If the field has non-zero mass, viz., it satisfies the equation of motion $(\Delta_{AdS} + m^2)\phi = 0$ with $m \neq 0$, then a tadpole of the form $\mathcal{L}_{\rm SUGRA} = \mathcal{L}_0 + \frac{C}{N}\phi + \cdots$ in the Lagrangian will only shift the expectation value of $\phi$ by an amount proportional to $\frac{1}{N}$ but otherwise the solution will still retain the $AdS$ symmetries. This will not be true for a massless field where now the equation of motion is $\Delta\phi + \frac{C}{N} = 0$. One concludes that if there are no massless invariant scalar fields, then any $\frac{1}{N}$-corrections to the equations of motion does not destabilize the solution

A parallel argument can be made in the terms of the field theory. The large N limit of a given quantum field theory is defined in terms of rescaled couplings such that in terms of these couplings the $\beta$-function is finite (i.e. $N$-independent). We will denote the vector of such rescaled couplings of the theory by $\mathbf{g}_t$. The statement of finite $\beta$ function is that the RG equation is

$$\dot{\mathbf{g}}_t = F_0(\mathbf{g}_t)\,, \tag{2.1}$$

without any explicit dependence on $N$. Taking into account $\frac{1}{N}$ correction we can expand the $\beta$ function by

$$\dot{\mathbf{g}}_t = F_0(\mathbf{g}_t) + \frac{1}{N}F_1(\mathbf{g}_t) + \frac{1}{N^2}F_2(\mathbf{g}_t) + \cdots . \tag{2.2}$$

A familiar example is that of the ‘t Hooft effective coupling in the large N limit of QCD. The relevant term in the $\beta$-function at finite N is

$$(\dot{g^2_{\rm YM}}) = b_0 N g^4_{\rm YM} + \cdots,$$

which implies that the $\beta$-function for the ‘t Hooft coupling $g^2_{\rm eff} = g^2 N$ is independent of N.

If the classical supergravity vacuum is such that the dual theory at large N is conformal, viz. the spacetime is of the type $AdS_p \times M_{11-p}$, then the functional form of $F_0$ is constrained to be

$$F_0(\mathbf{g}_t) = \mathbf{M}\cdot\,,\mathbf{g}_t,$$

where $\mathbf{M}$ is a matrix which encodes the dimensions of the operators around the fixed point at large $N$. We also relabeled the couplings such that the large $N$ fixed point is at $\mathbf{g}_t = 0$.

Concentrating on the invariant operators, if there are no marginal operators, the effect of the $\frac{1}{N}$ corrections to these operators will be small and controllable. The reason is that even when we take the $\frac{1}{N}$ corrections into account there still is a fixed point close by. Expanding $F_1(\mathbf{g}_t)$ around the large $N$ fixed point, the renormalization group equation, to leading order in $\frac{1}{N}$-correction and $\mathbf{g}_t$, is

$$\dot{\mathbf{g}}_t = \mathbf{M}\cdot\mathbf{g}_t + \frac{1}{N}F_1(0) + \cdots \tag{2.3}$$

and is solved by

$$\mathbf{g}_t^* = -\frac{1}{N}\,\mathbf{M}^{-1}\cdot F_1(0) + \cdots . \tag{2.4}$$

This is possible because under the assumption that there are no marginal operators the matrix $\mathbf{M}$ is invertible. Shifting the value of the fixed point in terms of the coupling is exactly analogous to giving small expectation value to non-zero mass fields.

Under more limited circumstances, the same line of argument can be applied to the case where the conformal field theory at the large N limit has marginal operators but not truly marginal ones.[1] For example, if the large N limit renormalization group flow near $g_t = 0$ is of the form

$$\dot{g}_t = a\, g_t^2 + \frac{1}{N}\, b + \cdots ,$$

where $a$ and $b$ are numbers, then there exists a fixed point at $g_t^{*2} = -\frac{1}{N}\frac{b}{a}$ provided $\frac{b}{a} < 0$.

We see that this kind of instability in non-supersymmetric theories occurs only when there are marginal operators and, in some cases, when there are marginal but no exactly marginal operators. In the case where in large N there are truely marginal operators then the effect of $F_1$ on the submanifold[2] of truely marginal deformations is pronounced. Since there is no flow on this submanifold at leading order in $1/N$, the sub-leading terms control the flow and there may simply not be a fixed point or it may be far away from our starting point. This will be the case with any typical vacua that is derived in string perturbation theory, where the dilaton will be associated with an invariant marginal operator. We will therefore discuss M-theory and specific vacua thereof and show that they do not have any marginal perturbations.

One might similarly obtain four-dimensional conformal field theory from Type IIB on $AdS_5 \times M^5$, but not in the perturbative regime. The generic problem, as we have explained above, is the tadpole for the dilaton field. However, there cannot be such a tadpole for the dilaton if it is fixed at the self-dual point under $SL(2,Z)$. In this case, the discrete gauge symmetry prevents generation of such a tadpole.

## 3. $d = 3$ non-supersymmetric CFT and $AdS_4 \times X_7$

The most extensive and varied list of M-theory comactification of the form $AdS_p \times X^{11-p}$ is available for the case $p = 4$. A large class of vacua is known and the stability (in the sense that all fields satisfy the Breitenlohner-Freedman unitarity bound [11]) of many of them has been analyzed. In particular, it has been shown that there are many non-supersymmetric yet stable vacua. To achieve the goal of this paper, we

[1]We are indebted to N. Seiberg for discussion of this point.
[2]in the space of coupling constants

are interested in examining the question whether these vacua have massless fields on $AdS_4$ space that are invariant under all the gauge symmetries of the supergravity theory. If a specific vacuum does not contain any massless, gauge-singlet scalar field, one would then conclude that the vacuum passes the tadpole-hurdle alluded in the previous section even after the $\frac{1}{N}$-corrections are taken into account. Such a vacuum may be dual to a non-supersymmetric conformal field theory.

A subset of the non-supersymmetric vacua of the form $AdS_4 \times X^7$ is obtained by "skew-whiffing" supersymmetric compactification vacua [12, 13]. The advantage of this method is that it guarantees the stability of the resulting non-supersymmetric vacua [14], at leading N. This is evidently so as the mass spectrum of the bosonic fields that might potentially cause an instability remains unchanged from that of the initial supersymmetric vacua and hence is stable. The disadvantage from our perspectives is that, in the vacua with more supersymmetries, one often encounters exactly marginal operators. According to our previous argument such operators, if exist and are invariant, would cause an instability to the non-supersymmetric vacua (those obtained by "skew whiffing") once $1/N$-corrections are made. On the other hand, exactly marginal operators become scarce for compactifications with less supersymmetries. We will henceforth focus on non-supersymmetric vacua obtained from "skew-whiffing" of compactifications with *smaller* supersymmetries.

### 3.1 Basics of "skew-whiffed" non-supersymmetric compactification vacua

Let us briefly review known facts about the mass spectrum of "skew-whiffed" compactification vacua. The "skew-whiffing" procedure is to reverse the orientation of the compactification manifold $X^7$. This is done by replacing $e^\mu_a$ by $-e^\mu_a$. The resulting background generated in this way is clearly a solution of the supergravity equations of motion. However, the amount of the residual supersymmetry preserved by the "skew-whiffed" vacuum changes in general. Indeed, apart from the most symmetric choice such as $X^7 = S^7$, no supersymmetry is left preserved after the orientation reversal of $X^7$ (in so far as the manifold $X^7$ is smooth).

Starting from the original supersymmetric vacuum, the mass spectrum of the "skew-whiffed" vacuum is obtained by interchanging the negative and the positive parts of the spectrum of the appropriate Laplacian operators on the compact manifold $X^7$ [14, 12] . Since the spectrum of these operators determines the masses of supergravity fields on $AdS_4$, the masses of some of the fields may change. For example, the spectrum of the spinors changes, corresponding to the fact that the new vacuum typically leaves out no residual supersymmetry at all.

Being non-supersymmetric, one might suspect that the "skew-whiffed" vacua generically contain some fields violating the unitarity bound and develop tachyonic instability. The supergravity fields that can potentially violate the unitarity bound are actually those with quantum numbers $J^P = 0^+$ on $AdS_4$. They arise from (traceless) deformations of the metric field on the compact manifold $X$. Fortunately, the



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mass spectrum of these fields does not change from that of the starting supersymmetric vacuum and hence the non-supersymmetric vacuum will remain stable as h well [14], at least for large $N$.

### 3.2 Generic problems

The argument given above holds only in the strict $N \to \infty$ limit. Clearly, the $\frac{1}{N}$-corrections will be different for the supersymmetric vacua and for their "skew-whiffed" ones. In the above discussion, we have utilized the underlying supersymmetry to argue that both the starting vacuum and the "skew-whiffed" one are stable at large N. However, the very same supersymmetry may also be a source of potentially dangerous operators. One needs to be aware of that the following problems might arise:

**Fields saturating unitarity bound may be present.** Since the $J^P = 0^+$ part of mass spectrum is the same as in the starting supersymmetric compactification, there might be fields that saturate the unitarity bound (and corresponding operator with dimensions-3/2 in the dual conformal field theory). The reason for this is that, in highly supersymmetric field theories, such operators occur quite commonly (for example, in three-dimensional $\mathcal{N} = 8$ gauge theory, the operator $trX^{\{i}X^jX^{k\}}$ has the scaling dimension 3/2). To avoid this problem, we will consider compactifications with less supersymmetries, for which generically we do not expect to find such operators. We then "skew-whiff" these compactifications and begin with the resulting non-supersymmetric vacua.

**(Exactly) Marginal operators may be present.** Since many of the dimensions in the "skew-whiffed" compactifications are inherited from the supersymmetric ones, there might be massless particles in the spectrum, inherited from exactly marginal operators in the original supersymmetric ones. As before, in supersymmetric gauge theories, it is quite common that (exactly) marginal scalar operators are present. Our main concern is whether a given compactification vacuum has any massless fields invariant under all supergravity gauge symmetries. While certainly a much more restricted class, it is again advantageous to focus on "skew-whiffing" of compactifications with less supersymmetries as they would not have exactly marginal operators in general.

Before we proceed to specific examples, let us identify which modes of the scalar field might be potentially dangerous in the sense that they will correspond to massless gauge singlet fields on the $AdS_4$. Since our foregoing discussion will be based on results from [12], we first note that the definition of mass in [12] is shifted from that we will be using momentarily. In [12] , the mass spectrum is defined according to $(\triangle - 8m^2 + M^2)S = 0$ (eq.(3.2.22) in [12] ), where $\triangle$ is the scalar Laplacian on the $AdS_4$ space, $m^2$ is a parameter associated with the compactification, and $M^2$ is the mass. In what follows, we will define the mass $\tilde{M}$ via $(\triangle + \tilde{M}^2)S = 0$, viz.,

$$\tilde{M}^2 = M^2 - 8m^2.$$

We are interested in identifying possible massless modes. The modes that are potentially dangerous to develop tadpoles are those of $J^P = 0^+$. Those of $J^P = 0^-$ cannot develop a tadpole as such a tadpole would break parity, viz., the symmetry $x^1 \to -x^1$, $C \to -C$, where $x^1$ is one of the spatial coordinates in $AdS_4$ and $C$ is the three-form potential of M-theory. Under our assumption of *calculability*, we expect this to be a symmetry of the dual conformal field theory. This symmetry then forbids generation of the tadpoles for the parity-odd fields.

Hence, potentially dangerous modes that might lead to massless states are

$$\begin{aligned} 0^{+(1)} &: \triangle_0 + 36m^2 - 12m^2(\triangle_0 + 9m^2)^{\frac{1}{2}} \\ 0^{+(2)} &: \triangle_L - 12m^2. \end{aligned}$$

The first will give rise to a dangerous mode for $\triangle_0 = 72m^2$ ($\triangle_0 = 0$ is omitted from the spectrum) and the second will give a dangerous mode for $\triangle_L = 12m^2$.

### 3.3 Examples

We will now present three examples of compactification of the form $AdS_4 \times X^7$, which are free from massless, gauge-singlet scalar fields. The full spectrum is known in the literature only for the first case. For the other two, the full spectrum is not known. As such, it is not clear whether there will be a field that saturates the unitarity bound. However since these will the "skew-shiffed" vacuum of a lower supersymmetry compactification, we do not expect such a scalar field in such compactifications to be present.

**Example 1:** $AdS_4 \times S^7/Z_2$**.** There are two types of $AdS_4 \times S^7/Z_2$ compactifications (even before taking into account different torsion classes [15, 16]) related by "skew-whiffing". M-theory on $S^7$ is dual to three-dimensional $\mathcal{N} = 8$ $SU(N)$ super-Yang Mills theory [17]. The supersymmetry charges would transform in one of the two spinor representations of $SO(8)$, say, $8_s$. Since we take a quotient by $Z_2$ which acts as $-1$ on the $8_v$ spinor, we can lift it to act as $+1$ on the $8_s$ spinor, in which case we still have $\mathcal{N} = 8$ supersymmetry. Alternatively, if it is lifted to act as $-1$ on the $8_s$ spinor, then supersymmetry is broken completely. Even though we have not casted it this way, this is basically equivalent to the "skew-whiffing" process.

It is easy to see that this compactification has no scalar field that can violate the unitarity bound. The only scalar field that lies at the unitarity bound [17] for $S^7$ is $trX^{\{i}X^jX^{k\}}$, but it is projected out by $Z_2$ quotient. One can also verify that there is no scalar field that can generate dangerous tadpoles: the only marginal operator with the $J^P = 0^+$ quantum number is in the symmetric traceless 6-tensor representation of $SO(8)$.



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**Example 2: the squashed 7-sphere.** It is known that there is one supersymmetric and one non-supersymmetric squashed 7-spheres [13]. The supersymmetric one has four-dimensional $\mathcal{N} = 1$ supersymmetry. The spectrum of Laplacians relevant for the $J^P = 0^+$ states is the same in both cases and is given by eqs. (8.4.2), (8.4.9) and (8.4.10) in [12]:

$$\triangle_0 = \frac{20}{9} m^2 C_G$$

$$\triangle_L = \frac{20}{9} m^2 \left( C_G + \frac{9}{5} \right) \qquad \text{or} \qquad \frac{20}{9} m^2 \left( C_G + \frac{8}{5} \pm \frac{2}{\sqrt{5}} \left( C_G + \frac{1}{20} \right)^{\frac{1}{2}} \right),$$

where $C_G = C_{SO(5)} + 3C_{SU(2)}$ . Since we interested in gauge-singlet fields, $C_G = 0$ and the masses of the relevant $J^P = 0^+$ obtained in this way turn out always non-zero.

**Example 3: $N(k,l)$.** The manifold $N(k,l)$ is defined [18] as a coset $[SU(3) \times U(1)]/[U(1) \times U(1)]$. The integers $k$ and $l$ parameterize different embeddings of the two $U(1)$'s into the maximal torus of $[SU(3) \times U(1)]$. For more details, the reader is referred to [18]. The supersymmetric compactifications on $N(k,l)$ have $\mathcal{N} = 1$ on $AdS_4$, and we will be interested in the "skew-whiffed" counterpart of them.

To examine whether there can be $SU(3) \times U(1)$ invariant massless field that might destabilize the vacuum, we again check possible massless fields in the spectrum of $J^P = 0^{+(1)}$ and $J^P = 0^{+(2)}$. For a symmetric space, however, $J^P = 0^{+(1)}$ does not lead to any dangerous modes. The reason is that these modes originate from the supergravity fields that are scalars on $X^7$ (hence eigenstates of the scalar Laplacian), and the only invariant mode is the constant mode for which $\Delta_0 = 0$. Such a constant mode, however, is not physical.

The analysis of the $0^{+(2)}$ mode is also straightforward. We are again interested only in modes which are invariant under $SU(3) \times U(1)$. The analysis of these modes is discussed in full detail in [18]. The fact that there is no invariant massless scalar field is essentially encoded in the computations of [18], where the authors find only a unique solution for every value of $k$ and $l$. Nevertheless, for completeness we briefly outline here the computation of the determinant of the mass matrix for these fluctuations. The result would be non-zero, indicating that there are no massless invariant fields.

To analyze the allowed metrics that preserve the $G = SU(3) \times U(1)$ symmetry, we first decompose the adjoint representation of this group into irreducible representations under the $ad(U(1) \times U(1))$. After we discard the subspace of $G$ which generates H, the remaining linear space is tangent to $X^7$ at the H-orbit that passes through the identity of $G$. The metric of this space splits into a sum of metrics on each of the $U(1) \times U(1)$ invariant subspaces. The freedom that we now have is to multiply each such component by an arbitrary number.

For $N(k,l) = [SU(3) \times U(1)]/[U(1) \times U(1)]$, this tangent space splits into four irreducible representations, three of them are of real-dimension 2 and one which is of real-dimension 1. These will be parameterized by indices $(a,b,\ldots)$, $(A,B,\ldots)$, $(\dot{A},\dot{B},\ldots)$ and $Z$, respectively. Denoting by $g^0$ the G-invariant metric (on $ad(g)$) the invariant metric on $M_7$ is of the form:

$$g_{ab}=\alpha g^0_{ab}\,,\qquad g_{ZZ}=\beta g^0_{ZZ}\,,\qquad g_{AB}=\gamma g^0_{AB}\,,\qquad g_{\dot{A}\dot{B}}=\delta g^0_{\dot{A}\dot{B}}\,. \tag{3.1}$$

The solution to Einstein's equations is given in [18], where it is written in terms of the variables

$$a=\frac{\alpha^2}{\delta^2}\,,\qquad b=\frac{\alpha^2}{\gamma^2}\,,\qquad u=+\frac{\alpha\gamma}{\beta\delta}\frac{3p+q}{\sqrt{2}}\,,\qquad v=-\frac{\alpha\delta}{\beta\gamma}\frac{3p+q}{\sqrt{2}}\,.$$

The solution is unique.

Once we have found the solution of Einstein equations, the allowed G-invariant fluctuations are fluctuations of $\alpha$, $\beta$, $\gamma$ and $\delta$ subject to the constraint that the total volume is invariant

$$2\frac{\triangle\alpha}{\alpha}+\frac{\triangle\beta}{\beta}+2\frac{\triangle\gamma}{\gamma}+2\frac{\triangle\delta}{\delta}=0\,.$$

Under small fluctuations of the three independent scale parameters $\alpha$, $\gamma$ and $\delta$, we find that

$$\begin{aligned}
\delta R^a_b &= \frac{\gamma^2\delta^2}{\alpha^2}\left[\frac{3}{2}ab+\frac{1}{4}\left(-1-a^2-b^2\right)-(av+bu)^2\right]\frac{\triangle\alpha}{\alpha}+ && (3.2)\\
&\quad+\left[\frac{1}{4}\left(1+b^2-a^2\right)-\frac{1}{2}(av+bu)^2\right]\frac{\triangle\gamma}{\gamma}+\left[\frac{1}{4}\left(1-b^2+a^2\right)-\frac{1}{2}(av+bu)^2\right]\frac{\triangle\delta}{\delta}\\
\delta R^Z_Z &= \frac{\gamma^2\delta^2}{\alpha^2}\left[(av+bu)^2+\frac{1}{2}u^2+\frac{1}{2}v^2\right]\frac{\triangle\alpha}{\alpha}+ && (3.3)\\
&\quad+\frac{\gamma^2\delta^2}{\alpha^2}\left[\frac{1}{2}(av+bu)^2+\frac{1}{2}v^2+u^2\right]\frac{\triangle\gamma}{\gamma}+\frac{\gamma^2\delta^2}{\alpha^2}\left[\frac{1}{2}(av+bu)^2+v^2+\frac{1}{2}u^2\right]\frac{\triangle\delta}{\delta}\\
\delta R^A_B &= \frac{\gamma^2\delta^2}{\alpha^2}\left[\frac{1}{4}\left(b^2+1-a^2\right)-\frac{1}{2}u^2\right]\frac{\triangle\alpha}{\alpha}+ && (3.4)\\
&\quad+\left[\frac{3}{2}a+\frac{1}{4}\left(-b^2-1-a^2\right)-u^2\right]\frac{\triangle\gamma}{\gamma}+\left[\frac{1}{4}\left(b^2-1+a^2\right)-\frac{1}{2}u^2\right]\frac{\triangle\delta}{\delta}\\
\delta R^{\dot{A}}_{\dot{B}} &= \frac{\gamma^2\delta^2}{\alpha^2}\left[\frac{1}{4}\left(a^2+1-b^2\right)-\frac{1}{2}v^2\right]\frac{\triangle\alpha}{\alpha}+ && (3.5)\\
&\quad+\left[\frac{1}{4}\left(a^2-1+b^2\right)-\frac{1}{2}v^2\right]\frac{\triangle\gamma}{\gamma}+\left[\frac{3}{2}b+\frac{1}{4}\left(-a^2-1-b^2\right)-v^2\right]\frac{\triangle\delta}{\delta}, && (3.6)
\end{aligned}$$

where only three out of the four variational equations are actually independent.

Choosing the three independent equations appropriately, we can cast these equations into a form

$$\delta R^\mu_\mu={M^\mu}_i\frac{\Delta V_i}{V_i}\,,$$

where $\mu=a,A,\dot{A}$ and $V_i=\alpha,\gamma,\delta$. From these expressions, we can deduce the equations of motion for the fluctuations $\Delta\alpha,\Delta\gamma$ and $\Delta\delta$. The result is that the mass matrix is the matrix $M_i^\mu$, up to multiplications by a non-degenerate matrix. Since we are interested in zero modes, this will not matter since we can evaluate the determinant of $M$, using the values of $\alpha, \gamma, \delta$ of the solution in [18]. The result is that there are no zero mass fields, i.e., there are no massless invariant fields that can acquire dangerous tadpoles.

Finally, a remark is in order. One might also prompt to explore stable higher-dimensional non-supersymmetric compactifications, especially, of the form $AdS_7 \times X^4$ and the corresponding dual conformal field theories. In the previous version of this letter, we have indicated $AdS_7 \times S^2 \times S^2$ might be a possible non-supersymmetric compactification that enables us to address the stability along the lines of section 2. It turned out that this vacuum is actually not a good starting point for $\frac{1}{N}$-expansion: the compactification is unstable under fluctuations for which the volume of the first $S^2$ shrinks and that of the second $S^2$ expands while keeping the total volume of $S^2 \times S^2$ fixed. Hence, barring the possibility of orbifold construction, it then appears that there is no stable non-supersymmetric compactification to $AdS_7$.

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